سلسلة تمارين 3 مع التصحيح

التمرين 1

انشر وبسط $\left(\sqrt{3}-5\right)^2$

التصحيح

$$\begin{aligned} E &= \left(\sqrt{3}-5\right)^2 \\ &= \sqrt{3}^2 + 5^2 - 2\times\sqrt{3}\times 5 \\ &= 3+25-10\sqrt{3} \\ &= 28-10\sqrt{3} \end{aligned}$$

التمرين 2

انشر وبسط $\left(2\sqrt{3}-1\right)\left(6-\sqrt{3}\right)$.

التصحيح

$$\begin{aligned} Z &= \left(2\sqrt{3}-1\right)\left(6-\sqrt{3}\right) \\ &= 2\sqrt{3}\times 6 - 2\sqrt{3}\times\sqrt{3} - 1\times 6 + 1\times\sqrt{3} \\ &= 12\sqrt{3} - 2\sqrt{3}^2 - 6 + \sqrt{3} \\ &= 12\sqrt{3} - 2\times 3 - 6 + \sqrt{3} \\ &= 13\sqrt{3} - 12 \end{aligned}$$

التمرين 3

1- احسب $B=\left(4-2\sqrt{3}\right)\left(4+2\sqrt{3}\right)$.

2- اكتب على شكل $a+b\sqrt{3}$ العددين

$C=\left(4-2\sqrt{3}\right)^2$ و $D=\frac{1}{4}\times\left(28-16\sqrt{3}\right)$.

التصحيح

-1

$$\begin{aligned} B &= \left(4-2\sqrt{3}\right)\left(4+2\sqrt{3}\right) \\ &= 4^2 - \left(2\sqrt{3}\right)^2 \\ &= 16 - 2^2\sqrt{3}^2 \\ &= 16 - 4\times 3 \\ &= 16 - 12 \\ &= 4 \end{aligned}$$

-2

$$\begin{aligned} C &= \left(4-2\sqrt{3}\right)^2 \\ &= 4^2 + \left(2\sqrt{3}\right)^2 - 2\times 4\times 2\sqrt{3} \\ &= 16 + 12 - 16\sqrt{3} \\ &= 28 - 16\sqrt{3} \end{aligned}$$

$$D = \frac{1}{4} \times \left(28 - 16\sqrt{3}\right)$$
$$= \frac{1}{4} \times 28 - \frac{1}{4} \times 16\sqrt{3}$$
$$= \frac{28}{4} - \frac{16}{4}\sqrt{3}$$
$$= 7 - 4\sqrt{3}$$

التمرين 4

اكتب التعابير الاتية على شكل $a + b\sqrt{2}$

$$\mathrm{E} = 5 + 6\sqrt{2}\left(3\sqrt{2} + 4\right) \qquad \mathrm{F} = \left(7\sqrt{2} - 4\right)^2$$

التصحيح

$$E = 5 + 6\sqrt{2}\left(3\sqrt{2} + 4\right)$$
$$= 5 + 6\sqrt{2} \times 3\sqrt{2} + 6\sqrt{2} \times 4$$
$$= 5 + 18\sqrt{2}^2 + 24\sqrt{2}$$
$$= 5 + 18 \times 2 + 24\sqrt{2}$$
$$= 5 + 36 + 24\sqrt{2}$$
$$= 41 + 24\sqrt{2}$$

$$F = \left(7\sqrt{2} - 4\right)^2$$
$$= \left(7\sqrt{2}\right)^2 + 4^2 - 2 \times 7\sqrt{2} \times 4$$
$$= 7^2\sqrt{2}^2 + 16 - 56\sqrt{2}$$
$$= 49 \times 2 + 16 - 56\sqrt{2}$$
$$= 94 + 16 - 56\sqrt{2}$$
$$= 110 - 56\sqrt{2}$$

التمرين 5

اكتب التعبير **E** على شكل $a + b\sqrt{5}$

$$: \mathrm{E} = \left(\sqrt{5} + \sqrt{3}\right)\left(\sqrt{5} - \sqrt{3}\right) - 8\sqrt{5}\left(\sqrt{5} - 1\right)$$

التصحيح

$$E = \left(\sqrt{5} + \sqrt{3}\right)\left(\sqrt{5} - \sqrt{3}\right) - 8\sqrt{5}\left(\sqrt{5} - 1\right)$$
$$E = \sqrt{5}^2 - \sqrt{3}^2 - 8\sqrt{5} \times \sqrt{5} + 8\sqrt{5} \times 1$$
$$E = 5 - 3 - 8 \times 5 + 8\sqrt{5}$$
$$E = -38 + 8\sqrt{5}$$

التمرين 6

1- بسط العددين $\sqrt{18}$ و $\sqrt{12}$

2- انشر وبسط $\left(10+4\sqrt{6}\right)\left(\sqrt{3}-\sqrt{2}\right)$

3- الجدول الاتي هل يمثل وضعية تناسبية

$\sqrt{3}+\sqrt{2}$	$10+4\sqrt{6}$
$\sqrt{3}-\sqrt{2}$	2

الجواب

-1

$$1)\ \sqrt{18}=\sqrt{9\times 2}=\sqrt{9}\times\sqrt{2}=3\sqrt{2}$$
$$\sqrt{12}=\sqrt{4\times 3}=\sqrt{4}\times\sqrt{3}=2\sqrt{3}$$

-2

$$\left(10+4\sqrt{6}\right)\left(\sqrt{3}-\sqrt{2}\right)=10\times\sqrt{3}-10\times\sqrt{2}+4\sqrt{6}\times\sqrt{3}-4\sqrt{6}\times\sqrt{2}$$
$$=10\sqrt{3}-10\sqrt{2}+4\sqrt{18}-4\sqrt{12}$$
$$=10\sqrt{3}-10\sqrt{2}+4\times 3\sqrt{2}-4\times 2\sqrt{3}$$
$$=10\sqrt{3}-8\sqrt{3}-10\sqrt{2}+12\sqrt{2}$$
$$=2\sqrt{3}+2\sqrt{2}$$

3- نعم الان

$$\left(10+4\sqrt{6}\right)\left(\sqrt{3}-\sqrt{2}\right)=2\sqrt{3}+2\sqrt{2}$$
$$2\times\left(\sqrt{3}+\sqrt{2}\right)=2\sqrt{3}+2\sqrt{2}$$

التمرين 7

-1 نعتبر التعبيرين $A=3+\sqrt{11}$ و $B=3-\sqrt{11}$.

احسب A^2 ; B^2 ; AB

-2

نعتبر $C=\sqrt{45}\times\sqrt{10}$ و $D=2\sqrt{50}+\sqrt{72}-\sqrt{2}$.

بسط ; C و D

الحل

$$A^2=\left(3+\sqrt{11}\right)^2$$
$$=3^2+\sqrt{11}^2+2\times 3\times\sqrt{11}$$
$$=9+11+6\sqrt{11}$$
$$=20+6\sqrt{11}$$

$$B^2=\left(3-\sqrt{11}\right)^2$$
$$=3^2+\sqrt{11}^2-2\times 3\times\sqrt{11}$$
$$=9+11-6\sqrt{11}$$
$$=20-6\sqrt{11}$$

$$A \times B = \left(3+\sqrt{11}\right)\left(3-\sqrt{11}\right)$$
$$= 3^2 - \sqrt{11}^2$$
$$= 9 - 11 = -2$$
$$C = \sqrt{45} \times \sqrt{10}$$
$$= \sqrt{9 \times 5 \times 5 \times 2}$$
$$= \sqrt{9} \times \sqrt{5^2} \times \sqrt{2}$$
$$= 3 \times 5 \times \sqrt{2}$$
$$= 15\sqrt{2}$$

$$D = 2\sqrt{50} + \sqrt{72} - \sqrt{2}$$
$$= 2\sqrt{25 \times 2} + \sqrt{36 \times 2} - \sqrt{2}$$
$$= 2\sqrt{25} \times \sqrt{2} + \sqrt{36} \times \sqrt{2} - \sqrt{2}$$
$$= 2 \times 5\sqrt{2} + 6\sqrt{2} - \sqrt{2}$$
$$= 10\sqrt{2} + 6\sqrt{2} - \sqrt{2}$$
$$= 15\sqrt{2}$$

C=D

التمرين 8

1- نعتبر التعبيرين $B = 3\sqrt{2} + 4$ و $A = 3\sqrt{2} - 4$

احسب A + B, A - B, A² et A × B

الحل

$$A + B = 3\sqrt{2} - 4 + 3\sqrt{2} + 4$$
$$A + B = 6\sqrt{2}$$

$$A - B = 3\sqrt{2} - 4 - (3\sqrt{2} + 4)$$
$$A - B = 3\sqrt{2} - 4 - 3\sqrt{2} - 4$$
$$A - B = -8$$

$$A^2 = \left(3\sqrt{2} - 4\right)^2$$
$$A^2 = \left(3\sqrt{2}\right)^2 + 4^2 - 2 \times 3\sqrt{2} \times 4$$
$$A^2 = 9 \times 2 + 16 - 24\sqrt{2}$$
$$A^2 = 34 - 24\sqrt{2}$$

$$A \times B = \left(3\sqrt{2} - 4\right) \times \left(3\sqrt{2} + 4\right)$$
$$A \times B = \left(3\sqrt{2}\right)^2 - 4^2$$
$$A \times B = 9 \times 2 - 16 = 2$$

التمرين 9

بسط التعبيرين

$$C = \frac{\sqrt{8}}{\sqrt{18}} \quad ; \quad D = \left(\sqrt{2} + \sqrt{8}\right)^2.$$

التصحيح :

$$C = \frac{\sqrt{8}}{\sqrt{18}} = \frac{\sqrt{4} \times \sqrt{2}}{\sqrt{9} \times \sqrt{2}} = \frac{2}{3}$$

$$\begin{aligned} D &= \left(\sqrt{2} + \sqrt{8}\right)^2 \\ &= \sqrt{2}^2 + \sqrt{8}^2 + 2\sqrt{2} \times \sqrt{8} \\ &= 2 + 8 + 2\sqrt{16} \\ &= 10 + 2 \times 4 \\ &= 18 \end{aligned}$$

او

$$\begin{aligned} D &= \left(\sqrt{2} + 2\sqrt{2}\right)^2 \\ &= \left(3\sqrt{2}\right)^2 \\ &= 3^2 \sqrt{2}^2 \\ &= 9 \times 2 = 18 \end{aligned}$$

التمرين 10

1- بسط $D = \sqrt{12} - \sqrt{75} - 2\sqrt{27}$

2- نعتبر

$$a = 3\sqrt{5} - 2\sqrt{11} \qquad b = 3\sqrt{5} + 2\sqrt{11}$$

احسب ab

الحل

$$\begin{aligned} D &= \sqrt{12} - \sqrt{75} - 2\sqrt{27} \\ &= \sqrt{4} \times \sqrt{3} - \sqrt{25} \times \sqrt{3} - 2\sqrt{9} \times \sqrt{3} \\ &= 2\sqrt{3} - 5\sqrt{3} - 2 \times 3\sqrt{3} \\ &= -9\sqrt{3} \end{aligned}$$

-2

$$\begin{aligned} a \times b &= \left(3\sqrt{5} - 2\sqrt{11}\right)\left(3\sqrt{5} + 2\sqrt{11}\right) \\ &= \left(3\sqrt{5}\right)^2 - \left(2\sqrt{11}\right)^2 \\ &= 3^2 \sqrt{5}^2 - 2^2 \sqrt{11}^2 \\ &= 9 \times 5 - 4 \times 11 \\ &= 45 - 44 = 1 \end{aligned}$$

التمرين 12

نعتبر

$A=\sqrt{48}+\sqrt{20}$ و $B=\sqrt{108}-\sqrt{45}$.

1- بسط A و B

2- بين ان AB عدد صحيح

التصحيح

-1

$$\begin{aligned} A &= \sqrt{48}+\sqrt{20} \\ &= \sqrt{16\times 3}+\sqrt{4\times 5} \\ &= \sqrt{16}\times\sqrt{3}+\sqrt{4}\times\sqrt{5} \\ &= 4\sqrt{3}+2\sqrt{5} \\ B &= \sqrt{108}-\sqrt{45} \\ &= \sqrt{36\times 3}-\sqrt{9\times 5} \\ &= \sqrt{36}\times\sqrt{3}-\sqrt{9}\times\sqrt{5} \\ &= 6\sqrt{3}-3\sqrt{5} \end{aligned}$$

-2

$$\begin{aligned} AB &= \left(4\sqrt{3}+2\sqrt{5}\right)\left(6\sqrt{3}-3\sqrt{5}\right) \\ &= 2\left(2\sqrt{3}+\sqrt{5}\right)3\left(2\sqrt{3}-\sqrt{5}\right) \\ &= 2\times 3\left(2\sqrt{3}+\sqrt{5}\right)\left(2\sqrt{3}-\sqrt{5}\right) \\ &= 6\left[\left(2\sqrt{3}\right)^2-\sqrt{5}^2\right] \\ &= 6\left[2^2\times 3-5\right] \\ &= 6\times 7 = 42 \end{aligned}$$

طريقة ثانية

$$\begin{aligned} AB &= \left(4\sqrt{3}+2\sqrt{5}\right)\left(6\sqrt{3}-3\sqrt{5}\right) \\ &= 4\sqrt{3}\times 6\sqrt{3}-4\sqrt{3}\times 3\sqrt{5}+2\sqrt{5}\times 6\sqrt{3}-2\sqrt{5}\times 3\sqrt{5} \\ &= 24\times 3-12\sqrt{15}+12\sqrt{15}-6\times 5 \\ &= 72-30 = 42 \end{aligned}$$

التمرين 11

1- انشر التعبير $A(x) = (2x + 1)(2x - 1)$

2 - احسب $A(x)$ إذا علمت أن $x=\sqrt{5}$

3- استنتج $20\,001 \times 19\,999$